

1. Write the equations (if possible) and explain the following terms as clear as possible.

- (a) Lorentz gauge and Coulomb gauge. (4%)
- (b) Gauge transformations and gauge freedom. (4%)
- (c) Lienard-Wiechert potentials. (4%)
- (d) The two postulates of the special relativity (4%)
- (e) Conserved quantity and invariant quantity. (4%)

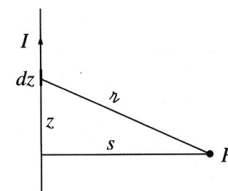
2. (a) The transformations between two inertial systems S and $\bar{S}$ are $\bar{x} = \gamma(x - vt)$ and $\bar{t} = \gamma(t - vx/c^2)$. Show that when $\Delta t = 0$, $\Delta x = \Delta \bar{x}/\gamma$; but when $\Delta \bar{t} = 0$, $\Delta \bar{x} = \Delta x/\gamma$. Explain why the length relations depend on the simultaneity. (10%)
- (b) Show that $(E^2 - c^2 B^2)$ is relativistically invariant. (10%)

3. Show that the retarded potential satisfy the Lorentz gauge condition. (20%)

[Hint: the retarded potentials $V(\mathbf{r}, t) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\mathbf{r}', t_r)}{r} d\tau'$ and $\mathbf{A}(\mathbf{r}, t) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}(\mathbf{r}', t_r)}{r} d\tau'$.]

4. (20%) An infinite straight wire carries a linearly increasing current: $I(t) = kt$ for $t > 0$.

- (a) Find the scalar and vector potentials. (10%)
- (b) Find the electric field generated. (10%)



5. The Maxwell equations can be written in terms of the field tensor.

$$F^{\mu\nu} = \begin{Bmatrix} 0 & E_x/c & E_y/c & E_z/c \\ -E_x/c & 0 & B_z & -B_y \\ -E_y/c & -B_z & 0 & B_x \\ -E_z/c & B_y & -B_x & 0 \end{Bmatrix} \quad [\text{Hint: } t^{\mu\nu} = \begin{Bmatrix} t^{00} & t^{01} & t^{02} & t^{03} \\ t^{10} & t^{11} & t^{12} & t^{13} \\ t^{20} & t^{21} & t^{22} & t^{23} \\ t^{30} & t^{31} & t^{32} & t^{33} \end{Bmatrix}].$$

- (a) Find the corresponding Maxwell's equation for $\frac{\partial F_{\mu\nu}}{\partial x^\lambda} + \frac{\partial F_{\lambda\mu}}{\partial x^\nu} + \frac{\partial F_{\nu\lambda}}{\partial x^\mu} = 0$, when $(\lambda, \mu, \nu) = (1, 2, 3)$. (10%)
- (b) Find the corresponding Maxwell's equation for $\frac{\partial F^{\mu\nu}}{\partial x^\nu} = \mu_0 J^\mu$, when $\mu = 1, 2$, and 3. (10%)

